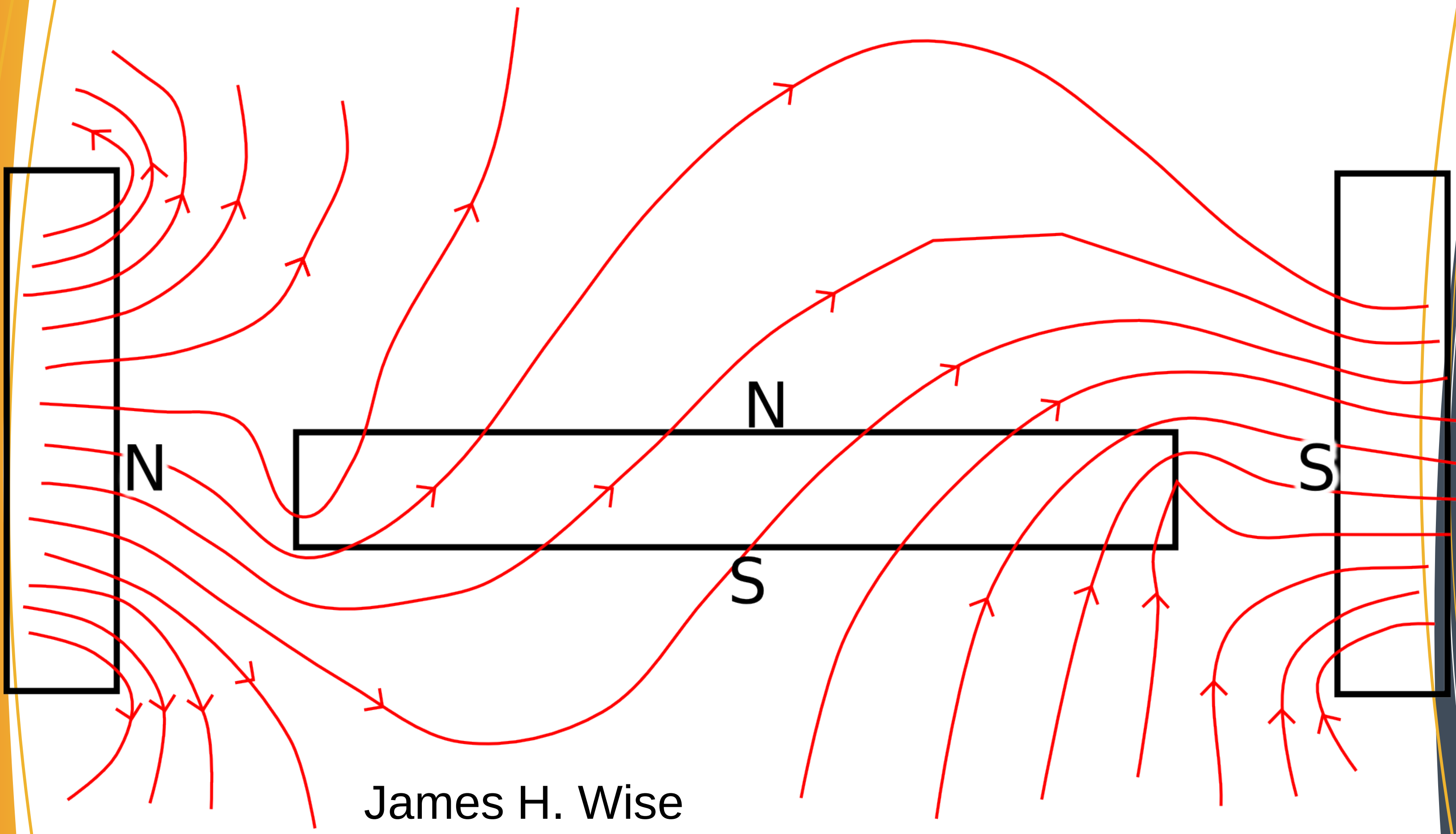
Calculating Magnetic Fields



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Magnetic Fields: What and Where?

Magnetic fields:

Sources:

- Appear around electric currents
- Surround magnetic materials

Properties:

- They have a direction
- They possess magnitude
- They are ***vector fields***

Reference for Definition

Field descriptions and behavior

See Chapters 1, 2 and 3 of "**The Feynman Lectures in Physics**", Vol.II by Feynman, Leighton and Sands, Addison Wesley,1964, ISBN 0 -201-2117-X-P .

Chapter 1 is my favorite starting point for thinking about fields. The next slide is derived from that.

Examples of Fields

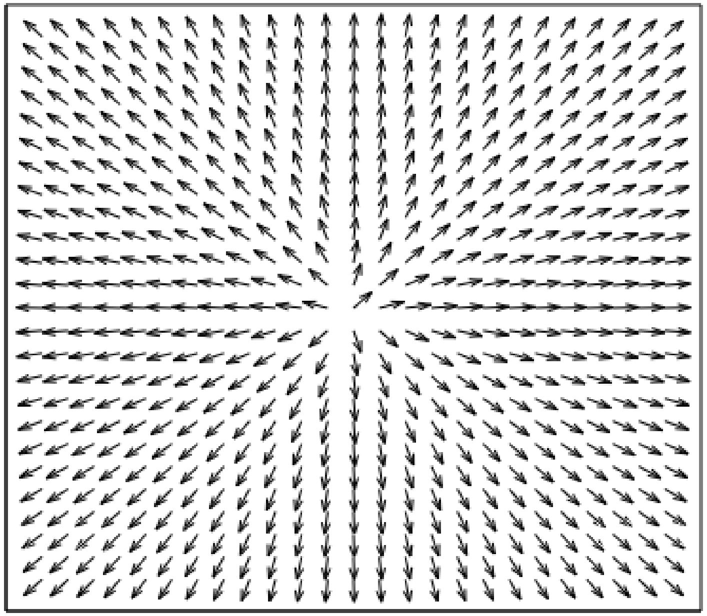
Fields:

- When a quantity varies with location in space, (x,y,z,t), it is said to be a field.
- Temperature around a heat source forms a field.
- As the temperature is only a single quantity associated with it is said to be a ***scalar field***
- Heat flow has a direction and thus 3 quantities that change with (x,y,z) and perhaps a time dependence
- Because it has 3 spatial components obeying the rules of vectors, it is said to be a ***vector field***.

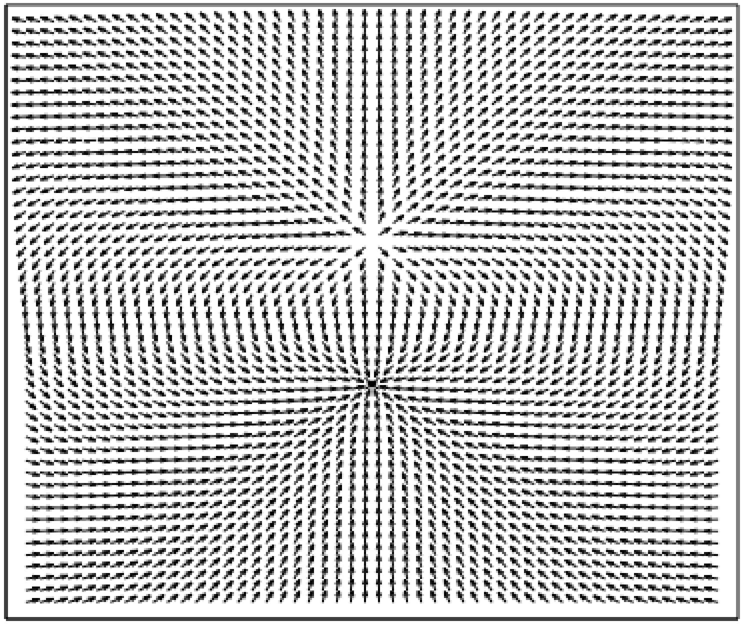
Magnetic Fields of Permanent Magnets

- The magnetic field was originally treated as originating from charges.
- Pole strength was interpreted as the amount of magnetic charge on the surface of a magnet If the charge distribution was known, the force outside the magnet could be computed correctly
- The charge model **does not** give the correct results for fields inside magnets.
- *The pole model is useful for gaining an intuitive view for qualitative results*.

Comparing Electric and Magnetic Fields



Electric Field- Mono Pole



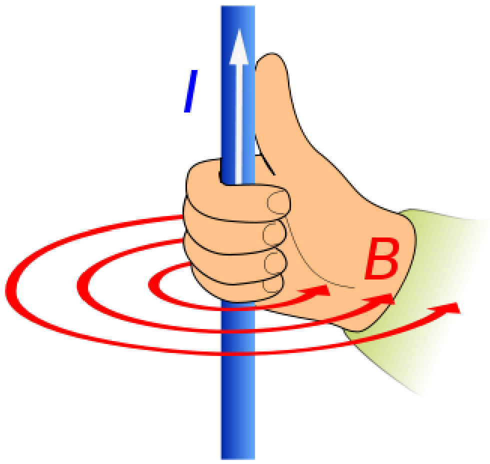
Magnet Field- Dipole
(at the least)

Electricity and Magnetic Fields

- In 1820 H.C Oersted discovered that a compass was affected by an electric current.
- Within days André Marie Ampère presented a set of results that described the forces between wires carrying electric currents.
- Also in 1820 Jean Baptiste Biot and Felix Savart presented the Biot Savart law that correctly portrayed the intensity and direction of magnetic fields around currents.

The Direction of Fields caused by Currents

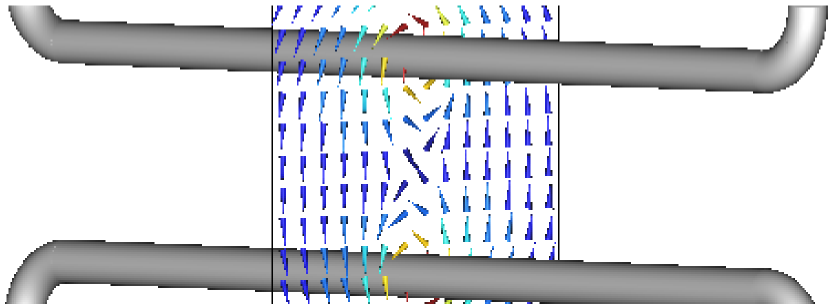
- Magnetic field lines form concentric circles around a cylindrical conductor.
- The direction of the field and current are related by a right hand rule



Graphic: Wikipedia Commons

Forces between current carrying conductors 1

- Ampere's experiments- Parallel currents attract



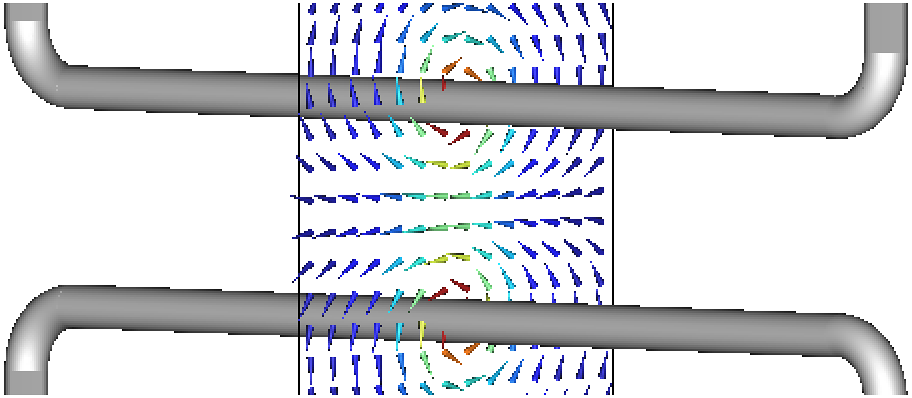
$$F_m = -\frac{2k_A I_1 I_2}{r}$$ Per unit length

Graphic: WMA-JW

Forces between current carrying conductors 2

Magnetic Fields (4):

- Ampere's experiments- Anti-parallel currents repel

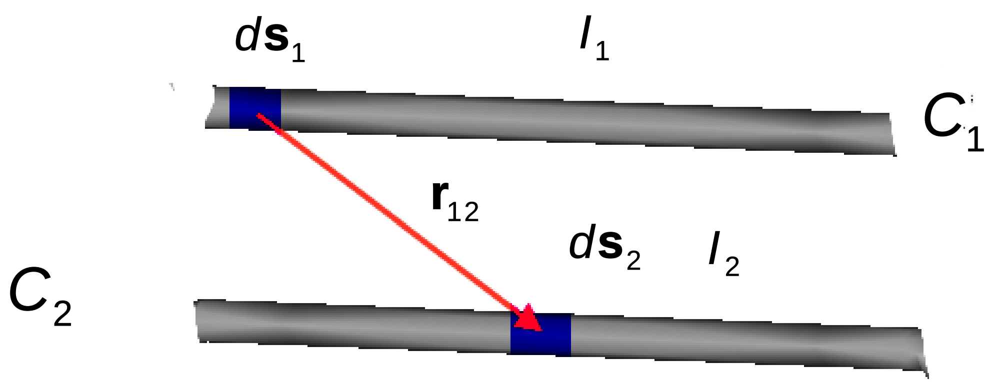


$$F_m = \frac{2k_A I_1 I_2}{r}$$ Per unit length

Graphic: WMA-JW

Forces between current carrying conductors 3

- Ampere's Force Law – General Expression



$$\mathbf{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{C1} \oint_{C2} \frac{d\mathbf{s}_2 \times (d\mathbf{s}_1 \times \hat{\mathbf{r}}_{12})}{|\mathbf{r}_{12}|^2}$$

Graphic: WMA-JW

The Biot-Savart Law 1

- The magnetic field generated by a current at a point r distant from a current filament is governed by the Biot-Savart Law:

$$\hat{\mathbf{r}}$$

$$\mathbf{B} = \frac{\mu_0 \mathbf{I}}{4\pi} * \oint \frac{\mathbf{dl} \times \hat{\mathbf{r}}}{r^2}$$

B = magnetic field
m_0= permeability of free space, $4\pi 10^{-7}$
I =current in amperes
dl = current element in filament
r = distance from the element to the field point
r = unit vector from element to field point

The Biot-Savart Law 2

- One important consequence of the Biot-Savart law is that the **B** field follows Gauss's law: the amount of flux leaving closed volume equals the amount of flux entering it.

$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

Magnetic Fields

Magnetic Fields:

- Amperes Circuital Law is another way of looking at the field. If the B field is integrated around a contour surrounding current(s):

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 \cdot I_{enc}$$

B = magnetic field
m_0= permeability of free space, $4\pi 10^{-7}$
$\mathbf{I}_{enc}$ =current enclosed by the contour in amperes
dl = element of path around the conductors

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Magnetic Fields

Magnetic fields:

- **H**-field:
  - Prior to measuring magnetic materials in fields, there was a field between permanent magnets, apparently issuing from magnetic charges, so a field **H** was defined. It is quantitatively different from **B**
  - Biot Savart and Ampere Laws can be applied to H -
  - We remove the permeability of free space, m_0

$$\mathbf{H} = \frac{\mathbf{I}}{4\pi} * \oint \frac{\mathbf{dl} \times \hat{\mathbf{r}}}{r^2}$$

$$\oint \mathbf{H} \cdot \mathbf{dl} = I_{enc}$$

Magnetic Fields

Magnetic Fields :

- Amperes Circuital Law is another way of looking at the field. It is more commonly used in terms of an **H** field – the field due to currents only
- Ampere’s law is used regularly in magnetic design. As we move through the parts of a simple magnetic circuit where H is parallel to dl in every part of the path, we will use this law in a form:

$$H_1 l_1 + H_2 l_2 + \cdots + H_n l_n = N I_{enc}$$

Faraday's Law of Induction

- In 1831 Michael Faraday published the rule we now call Faraday's law of induction.
- Faraday's law states that *the electromotive force (emf) induced in a circuit is equal to the rate of change of flux through the circuit*:

- Mathematically formulated by James Clerk Maxwell:

$$|\mathbf{E}| = \left|\frac{d\Phi}{dt}\right|$$

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Lenz’s Law

- In 1834 Heinrich Lenz published Lenz’s Law
- *An induced current is always in a direction that opposes the change that causes it*.
- Accordingly Faraday’s Law is modified

$$E = -\frac{d\Phi}{dt}$$

Defining Units For Magnetic Fields

- Ampere's Force Law –We can now define a unit of current, the ***ampere*** because we can measure force in newtons between wires carrying currents
- Biot - Savart defines the magnitude and direction of fields produced by currents- and units for fields. The units are **Tesla.**
- Gauss's law defines magnetic flux – units of measurement. The unit of **flux** is a **Weber**
- Amperes circuital theorem will allows us to define magneto motive forces in circuits using **Amperes-turns**
- Faraday's and Lenz's laws give methods of measuring magnetic fields and relating **volts**, **seconds** and **Teslas**, **Webers**, and **meters.**

Magnetic fields and Permanent Magnets

- Ampere chose to view magnetism in materials as arising from electrical currents within the material.
- The currents within the material sum to give a current circulation on the surface of the material
- The material is still magnetic if the outer layer is removed so the currents must be viewed as distributed through the material.
- No single magnetic charge was ever found- magnetic materials always have two poles, like a ring of current
- The Amperian model predicts the correct field inside and outside the magnet.

Three fields are used to describe magnetic behavior

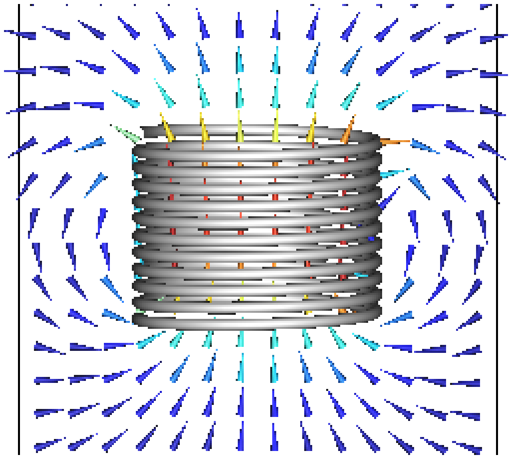
- **B-field:**
  - Is the response of a material to the application of a magnetic field
- **M-field**
  - Is the magnetic field produced by a material
  - M is the magnetization of the material
- **H – field**
  - Is a modification of the B field made by the M field
  - H is also the field produced by currents alone absent the effects of media

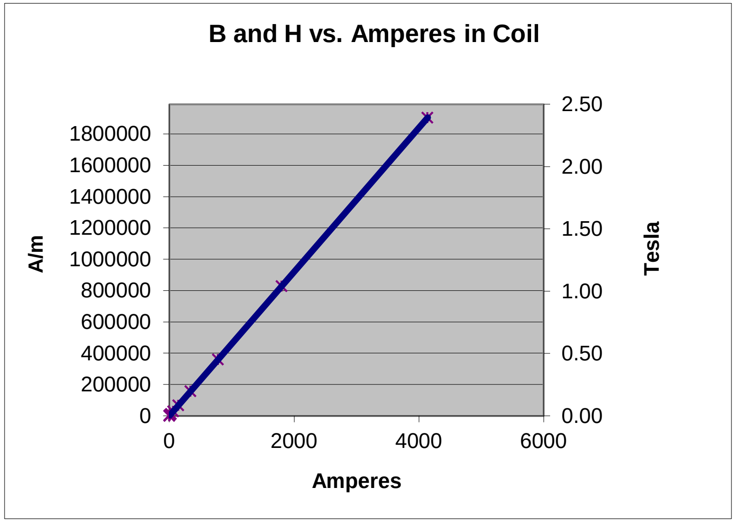
B-field

- The units for the B field are ***webers/ m²***
- The weber is derived from Faraday's law
- Faraday's law states that the electromotive force (emf) induced in a circuit is equal to the rate of change of flux through the circuit.
- "1 ***weber*** is equal to the amount of flux that when reduced uniformly to zero in 1 second produces an emf of 1 volt in a 1 turn coil." [1]
- The weber can also be defined from Ampere's force law (see the later definition of **tesla** below)

B in a coil

- Put current in varying amounts through a coil in vacuum
- Measure the magnetic flux through a coil of known area in the center of the coil

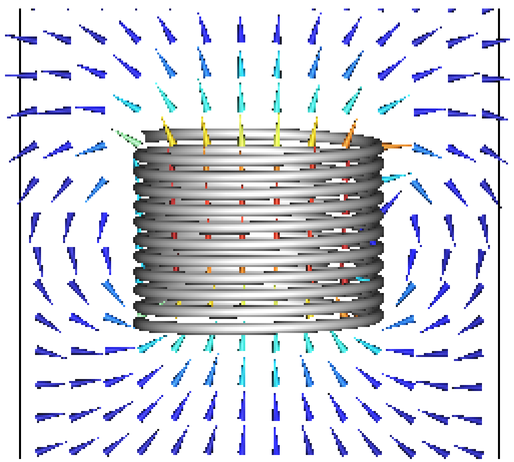


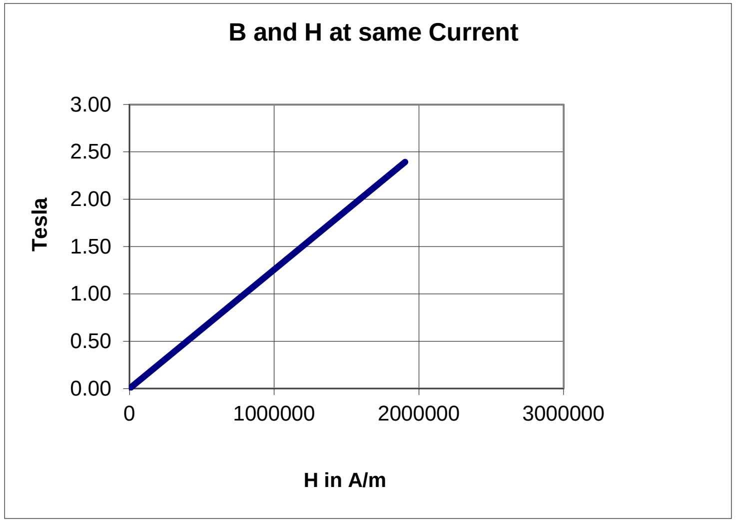


B and H in a coil

- The B line is from the application of Biot-Savart deriving B. H is from applying it to H

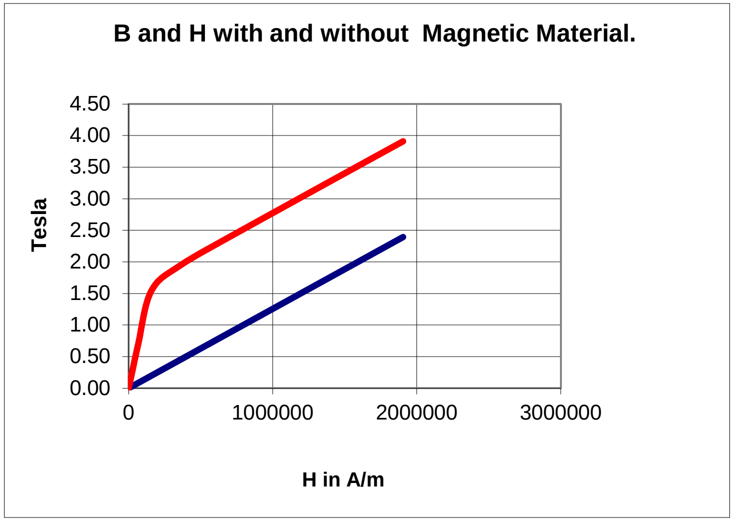
$$B = 4 \cdot \pi \cdot 10^{-7} * H$$





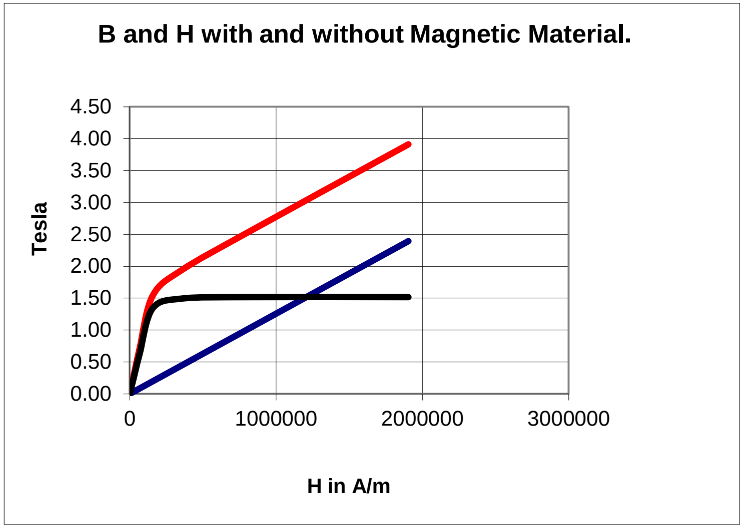
B and H in materials

- We place a very long piece of steel in the coil
- The steel adds something to the B field



B and H in materials 1

- The difference is M the magnetization of the material



H Field

- We have to define an **H** field to break the **B** field into well understood components
- Without a material present there is still something present because we can measure **B**
- When a magnetic material is present we get a an addition to the B field we sense without it.

For more information on calculating magnetic fields, please call
Jim Wise at 219-548-3799 or email to jim@allianceorg.com